MAHARAJA RANJIT SINGH PUNJAB TECHNICAL UNIVERSITY BATHINDA (Pb) - 151001

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Discipline: MATHEMATICS (Faculty of Sciences)

3rd PhD ENTRANCE TEST (PET-2018)

Roll No: _____ Date: 3rd June 2018 Signature of the Candidate._____

Q.1 If B is a non-singular matrix and A is a square matrix, then $|B^{-1}AB| =$

a) $|B|$ b) $|A|$ c) $|A^{-1}|$ d) $|B^{-1}|$

Q.2 Find inverse Laplace Transformation of: $\frac{s^2+1}{s(s+1)(s+2)}$

a) $\frac{1}{2} - 2e^{-t} + \frac{5}{2}e^{-2t}$ b) $\frac{1}{2} - 2e^{-2t} + \frac{5}{2}e^{-t}$ c) $\frac{1}{2} + 2e^{-t} + \frac{5}{2}e^{-2t}$ d) None of these

Q.3 If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then $|\text{adj (adj } A)| =$

a) 14^2 b) 14 c) 14^4 d) None of these

Q.4 The general solution of partial differential equation $\frac{\partial^4 z}{\partial x^4} - \frac{\partial^4 z}{\partial y^4} = 0$ is:

a) $z = f_1(y+x) + f_2(y-x) + f_3(y+ix) + f_4(y-ix)$
b) $z = f_1(y+x) + f_2(y-x) + f_3(y^2+x^2) + f_4(y^2-x^2)$
c) Both (a) and (b)
d) None of these

Q.5 If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I = O$, then $k =$

a) -7 b) 7 c) 3 d) 5

Q.6 If X has a binomial distribution with parameters n and p, where n is an integer greater than 1 and p is the probability of success. If $P(X=0) = P(X=1)$, then $p =$

a) $\frac{1}{n-1}$ b) $\frac{n}{n-1}$ c) $\frac{1}{n+1}$ d) $\frac{n}{n+1}$

Q.7 Let G be a group of order 25. Then:

a) G is abelian b) G is cyclic c) G is non-abelian d) Centre of G has order 7

Q.8 For which of the following distributions, the weak law of large numbers does not hold?

a) Normal b) Beta c) Cauchy d) Gamma

Q.9 The maximum no. of linear independent solutions of differential equation $\frac{d^3y}{dx^3} = 0$ with the condition $y(0) = 1$ is:

a) 1 b) 2 c) 3 d) 4

Q.10 In answering a question on a multiple-choice test, a student either knows the answer or guesses. Let p be the probability that she knows the answer and $1 - p$ the probability that she guesses. Assume that a student who guesses at the answer will be correct with probability $1/m$, where m is the number of multiple-choice alternatives. What is the probability that a student knew the answer to a question given that she answered it correctly?

a) $\frac{p}{p + \frac{1}{m}(1-p)}$ b) $\frac{1}{1 + \frac{1}{m}\left(\frac{1}{p} - 1\right)}$ c) Both (a) and (b) d) None of the above

Q.11 Let $u(x, y) = 2xy$ for all real x and y. Then a function $v(x, y)$, so that $f(z) = u(x, y) + iv(x, y)$ is analytic, is:

a) $y^2 - x^2 + c$ b) $x^3 - y^3 + c$ c) $y^2 + x^2 + c$ d) None of these

Q.12 The coefficient of $\frac{1}{z}$ in the expansion of $\log\left(\frac{z}{z-1}\right)$ valid in $|z| > 1$, is:

a) -1 b) 1 c) $\frac{1}{2}$ d) $-\frac{1}{2}$

Q.13 The approximate eigen value of the matrix

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

obtained after 2 iterations of power method, with the initial vector $[0,0,1]^T$ is:

a) 7.768 b) 9.468 c) 10.548 d) 11.529

Q.14 The bilinear transformation w which maps the points $1, i, -1$ in the z − plane onto the points $i, 0, -i$ in w − plane is:

a) $\frac{z-1}{z+i}$ b) $\frac{1+iz}{1-iz}$ c) $\frac{z-i}{z-1}$ d) $\frac{z+i}{z-i}$

Q.15 Let C be a circle enclosing the origin and oriented counter clockwise. Then $\oint_C \frac{\sin z}{z^2}\, dz =$

a) $2\pi i$ b) $-2\pi i$ c) 0 d) None of these

Q.16 Let L be a line segment from $(1,1,1)$ to $(2,2,2)$.

Then $\int_L 3x^2y^3z^3dx + 3y^2x^3z^3dy + 3z^2x^3y^3dz$ is:

a) 63 b) 513 c) 511 d) 7

Q.17 Let $A = \{1,2,3\}$ and $R = \{(1,1), (1,3), (3,1), (2,2), (2,1), (3,3)\}$
Then the relation R on A is:

a) Reflexive and Symmetric b) Symmetric and Transitive c) Reflexive only d) Equivalence Relation

Q.18 From the Sturm Liouville problem $y'' + \lambda y = 0$ with $y(0) = 0$ and $y(\pi) = 0$, the eigen values, λ, satisfy:

a) $\frac{n^2}{4}$ b) $\frac{(2n-1)^2\pi^2}{4}$ c) $\frac{(2n+1)^2}{4}$ d) n^2

Q.19 Differential equation $(3\,a^2x^2 + by\sin 2x)dx + (2\sin^2 x + 3ay^2)dy = 0$ is exact for:

a) $a = 3,\ b = 2$ b) $a = 3,\ b = 4$ c) $a = 2,\ b = 3$ d) $a = 2,\ b = 5$

Q.20 In the group $(Z, +)$, the subgroup generated by 2 and 7 is:

a) Z b) $5Z$ c) $9Z$ d) $14Z$

Q.21 Consider wave equation: $\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi, t > 0$, with $u(0,t) = u(\pi,t) = 0$, $u(x,0) = 3\sin x$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0$. Then $u\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is:

a) 2 b) 1 c) 0 d) 3

Q.22 The partial differential equation $2\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = 0$, is:

a) Elliptic b) Parabolic c) Hyperbolic d) None

Q.23 The differential equation whose linear independent solutions are $\cos 2x,\ \sin 2x,\ e^{-2x}$ is:

a) $(D^3 + 2D^2 + 4D + 8)y = 0$ b) $(D^3 - 2D^2 + 4D + 4)y = 0$
c) $(D^3 - D^2 + D - 1)y = 0$ d) $(D^3 + D^2 - D + 1)y = 0$
(Where $= \frac{dy}{dx}$)

Q.24 The polynomial $f(x) = x^5 + 5$ is:

a) Irreducible over C b) Irreducible over R c) Irreducible over Q d) Not irreducible over Q
(Where Q denotes the field of rational no.)

Q.25 Let $y(x)$ be the solution of initial value problem:
$y''' - y'' + 4y' - 4y = 0, y(0) = y'(0) = 2,\ y''(0) = 0$. Then the value of $y(\pi/2) =$

a) $\frac{1}{5}(4e^{\pi/2} - 6)$ b) $\frac{1}{5}(6e^{\pi/2} - 4)$ c) $\frac{1}{5}(8e^{\pi/2} - 2)$ d) $\frac{1}{5}(2e^{\pi/2} - 8)$

Q.26 No. of irreducible quadratic polynomials over the field of two elements F_2 is:

a) 3 b) 2 c) 1 d) 0

Q.27 Laplace Transformation of $\sin t\ u(t-\pi)$ is:

a) $-\frac{e^{-\pi s}\, s}{s^2+1}$ b) $-\frac{e^{-\pi s}\, s}{s^2+\pi^2}$ c) $-\frac{e^{-\pi s}}{s^2+1}$ d) $\frac{1}{s^2+1}$

Where $u(t)$ is the unit step function

Q.28 The function defined as follows: $f(x) = \begin{cases} x^3; & x^2 < 1 \\ x\,; & x^2 \geq 1 \end{cases}$ is

a) Discontinuous at $x = 1$
b) Differentiable at $x = 1$
c) Continuous but not differentiable at $x = 1$
d) None of these

Q.29 While solving the equation $x^3 - 7x^2 + 16x - 12 = 0$ using the Newton-Raphson method with the initial guess of a root as 1, the value of the root, after one iteration is:

a) 1.40 b) 0.60 c) 1.33 d) 0.67

Q.30 Let y be the solution of the initial value problem $\frac{dy}{dx} = (y^2 + x); y(0) = 1$

Using Taylor series method of order-2 with the step size $h = 0.1$, the approximate value of $y(0.1)$ is:

a) 1.115 b) 1.215 c) 1.315 d) 1.415

Q.31 The eigen value λ of integral equation $y(x) = \lambda \int_0^1 (2xt - 4x^2) y(t)dt$ is:

a) $\lambda = 3$ b) $\lambda = -3$ c) $\lambda = 2$ d) $\lambda = -2$

Q.32 The differential equation corresponding to integral equation $y(x) = 1 + \int_0^1 (x + t) y(t)dt$ is:

a) $y''(x) - 2xy'(x) - 3y(x) = 0, y(0) = 1, y'(0) = 0$
b) $y''(x) - 5xy'(x) - 7y(x) = 0, y(0) = 1, y'(0) = 0$
c) $y''(x) - 5y'(x) + 7y(x) = 0, y(0) = 1, y'(0) = 0$
d) None of these

Q.33 The integral equation: $y(x) = 1 + \lambda \int_0^1 xt\, y(t)dt$ is solved by the method of successive approximations. Starting with initial approximation $y_0(x) = 1$, the second approximation, $y_2(x)$, is given by:

a) $1 + \frac{\lambda x}{3} + \frac{\lambda^3 x}{5}$ b) $1 + \frac{\lambda x}{2} + \frac{\lambda^2 x}{6}$

c) $1 + \frac{\lambda^2 x}{2} + \frac{\lambda^4 x}{4}$ d) None of these

Q.34 If $g: [1,2] \to R$ is a non-negative Riemann-integrable function such that

$$\int_1^2 \frac{g(x)}{x^2} dx = k \int_1^2 g(x)dx \neq 0$$ Then k belongs to the interval

a) $\left[0, \frac{1}{4}\right]$ b) $\left[\frac{1}{4}, 1\right]$ c) $\left[\frac{1}{4}, 0\right]$ d) None of these

Q.35 The set $X = R$ with metric

$$d(x,y) = \frac{|x-y|}{1+|x-y|}$$ is:

a) Bounded but not compact b) Bounded but not complete
c) Complete but not bounded d) Compact but not complete

Q.36 The series:

$$\sum_{n=1}^{\infty} \frac{\sin(x^2+nx)}{n(n+2)}, \forall x \in R$$ is:

a) Uniformly but not absolutely convergent b) Uniformly and absolutely convergent
c) Neither absolutely nor uniformly convergent d) Absolutely but not uniformly convergent

Q.37 A uniformly continuous function is:

a) Measureable b) Not measureable c) Measureable and simple d) Integrable and simple

Q.38 Consider the following improper integrals

$$I = \int_1^{\infty} \frac{dt}{1+t^2}$$ and $$J = \int_1^{\infty} \frac{dt}{(1+t^2)^{3/2}}$$ Then:

a) I converges but not J
b) J converges but not I
c) Both I & J converge
d) Neither I nor J converges

Q.39 Consider R^2 with usual topology. The complement of $N \times N$ is:

a) Connected but not open b) Open but not connected
c) Both open and connected d) Neither open nor connected

Q.40 A wire of length l and mass m is bent in the form of a rectangle ABCD with AB : BC $= 2:1$. The moment of inertia of this wire frame about the side BC is:

a) $(11/252)ml^2$ b) $(8/203)ml^2$ c) $(5/136)ml^2$ d) $(7/162)\, ml^2$

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